Teaching London Computing

A Level Computer Science

Topic 4: Functions and Recursion

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Aims

- Recursion the idea
  - Problem solving by divide and conquer
- Revision of functions
- Programming with recursion

Recursion

Recursion is a fundamental idea

Example: Factorial

- 5! = 5 * 4 * 3 * 2 * 1
- Factorial can be defined using factorial:

Is defined as

factorial(N) = N * factorial(N-1)

- But is this useful?

1. Base Case: factorial(0) = 1
2. Recursive case:
   factorial(N) = N * factorial(N-1), provided N > 0

Factorial Example

1. Base Case: factorial(0) = 1
2. Recursive case:
   factorial(N) = N * factorial(N-1), provided N > 0

factorial(4) = 4 * factorial(3)
= 4 * 3 * factorial(2)
= 4 * 3 * 2 * factorial(1)
= 4 * 3 * 2 * 1 * 1

Recursion

- Define something in terms of itself
  - Recursive case – smaller instance of the problem
  - Base case – smallest instance
- Some algorithms are elegantly stated recursively
- Recursive implementation
  - factR calls factR – recursive call
- Divide and conquer

Binary Search (Again)

- Search a sorted array – is E in the array?

- Base case:
  - empty array or E found
- Recursive case:
  - Compare E with the middle element
  - If E smaller, search the left half
  - If E larger, search the right half

Recursive Sorting

- Concept
  - Split array in two halves, sort each half
  - Combine two sorted arrays
  - Single item sorted (base case)

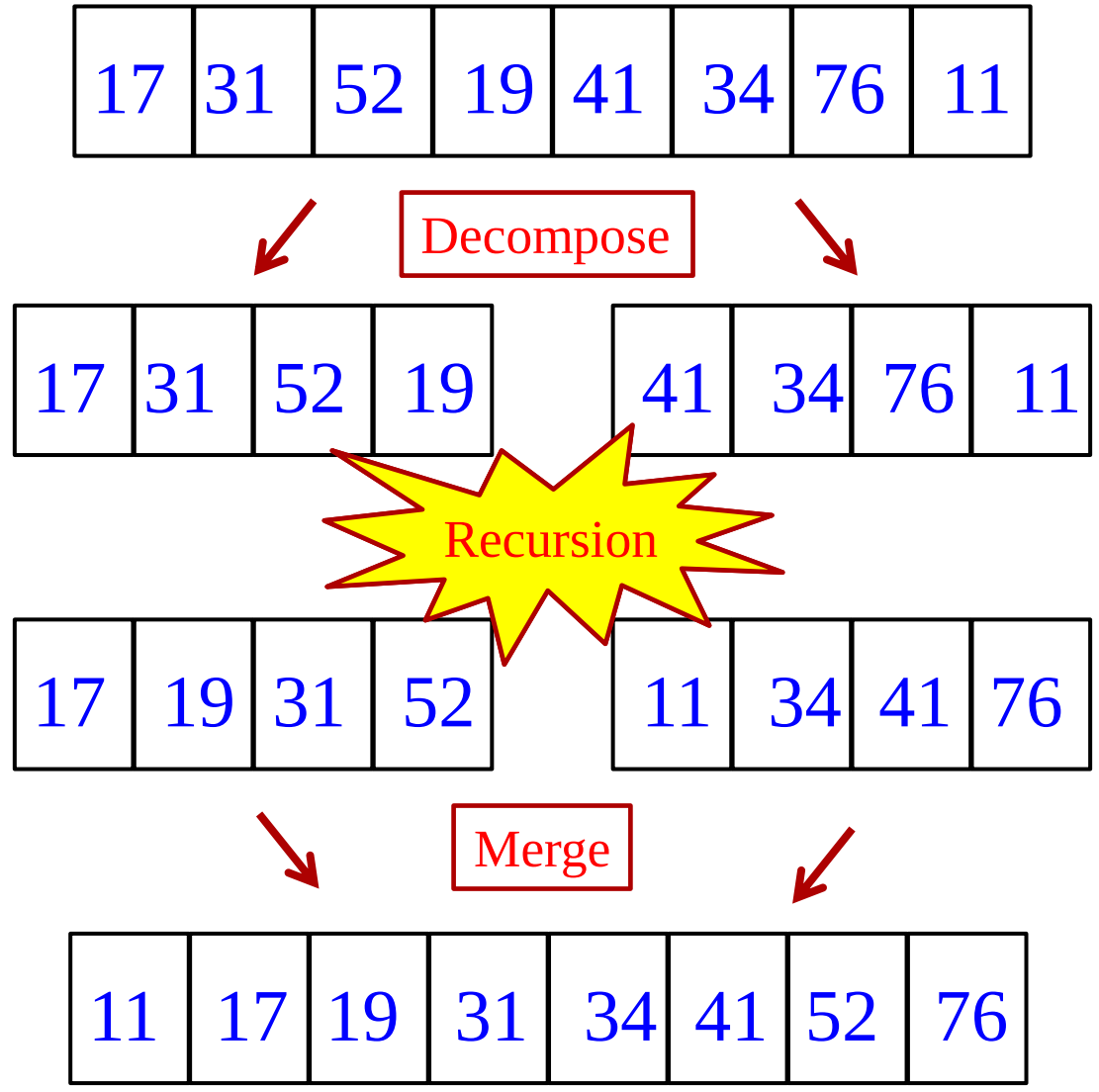
- Two algorithms
- Merge sort
  - Halve array – merge two sorted lists
- Quicksort
  - Partition array – combine easy

Merge Sort – Insight

- How could we share out sorting between two people?
  - Half it and sort each half
  - Merge the two sorted lists

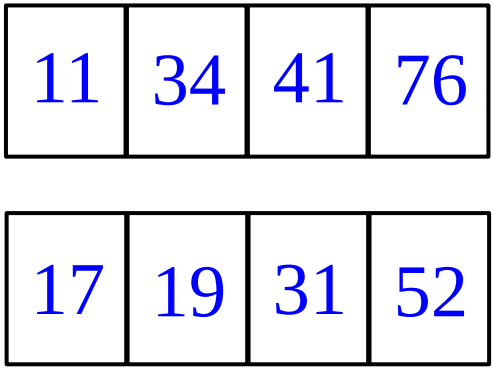
  - When there is only a single entry – it is sorted

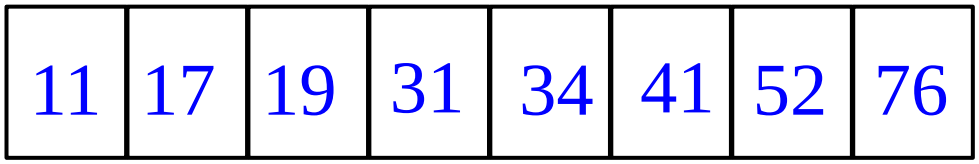
Merge Sort


17 31 52 19 41 34 76 11
Decompose
17 31 52 19
41 34 76 11
Recursion
17 19 31 52
11 34 41 76
Merge
11 17 19 31 34 41 52 76

Merging

- Work done in the merge
- Repeatedly select smallest



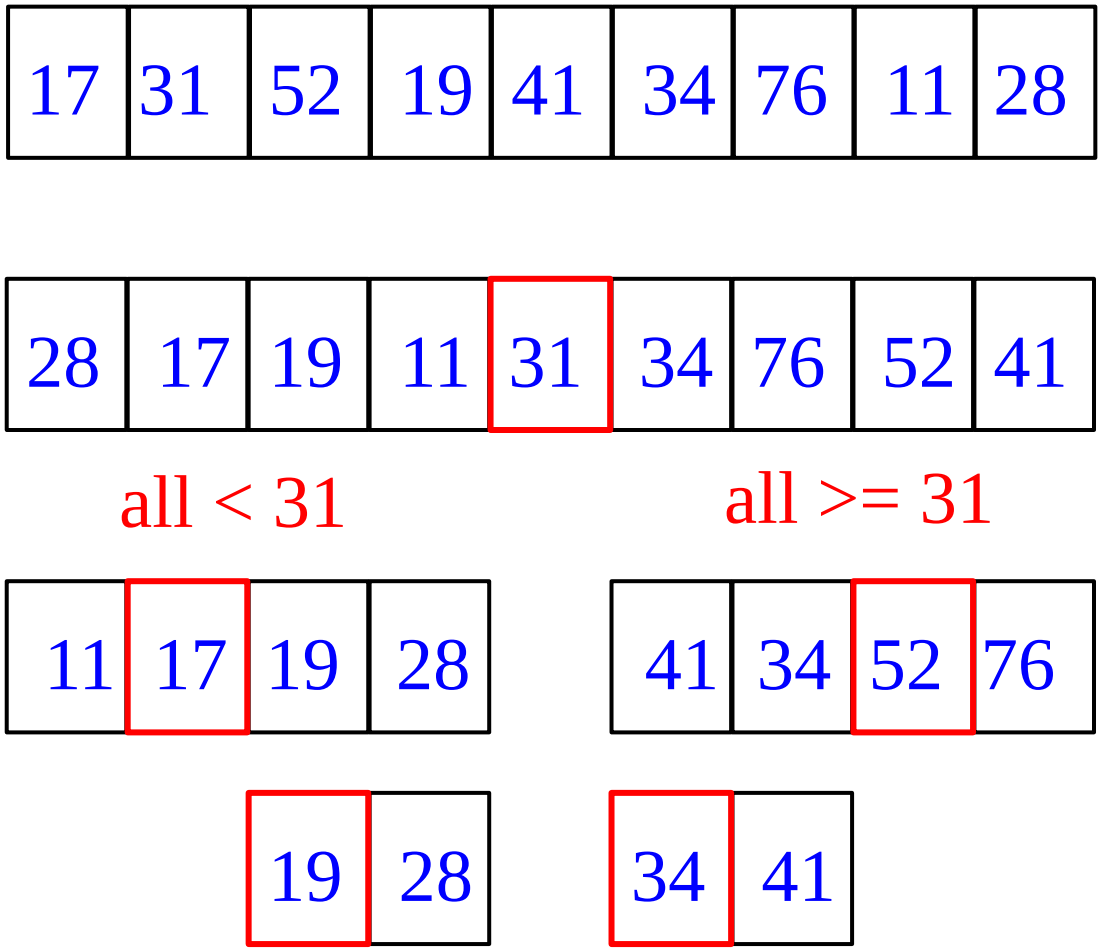


Quicksort – Insight

- How could we share out sorting between two people?
  - Choose a value V
  - Give first person all values < V
  - Give second person all values > V

  - When there is only a single entry – it is sorted

Quicksort Example



Quicksort Description

- Choose a **pivot value**
- Partition the array
  - Values less than the pivot to the left
  - The pivot
  - Values greater than the pivot to the right
- Repeat process on each partition
- … until partition has no more than one value

- **Work done in partition**

Exercise 1.1-1.2

- Demonstrate merge sort using playing cards. Can you make the use of recursion clear?
- Also try Quicksort

Properties

- Merge sort
  - O(N log N) – same as quick sort but extra space
  - Stable

http://www.sorting-algorithms.com/merge-sort

- Quicksort
  - More efficient: O(N logN)
  - Not stable

http://www.sorting-algorithms.com/quick-sort

Functions

To write recursive programs we need a good understanding of functions

Why Functions?

- Code organisation
  - Functions allow code to be organised in parts
  - Top-down development
- Code reuse
  - The library
  - Functions with parameters: existing code, your data
- Recursion

Elements of Functions

- Name and definition

  Issue: function must be called

- Parameters and return

  Issue: return versus print

- Local variables and scope

  Issue: where are the variables

Simple Functions

- No parameters

```
def greet():
    print("Hello")
```

- No return
- Issue
  - A function must print or return
  - Return and print are confused

```
def greetByName(name):
    print("Hello", name)
```

- Suggestions
  - Always use return
  - Avoid print

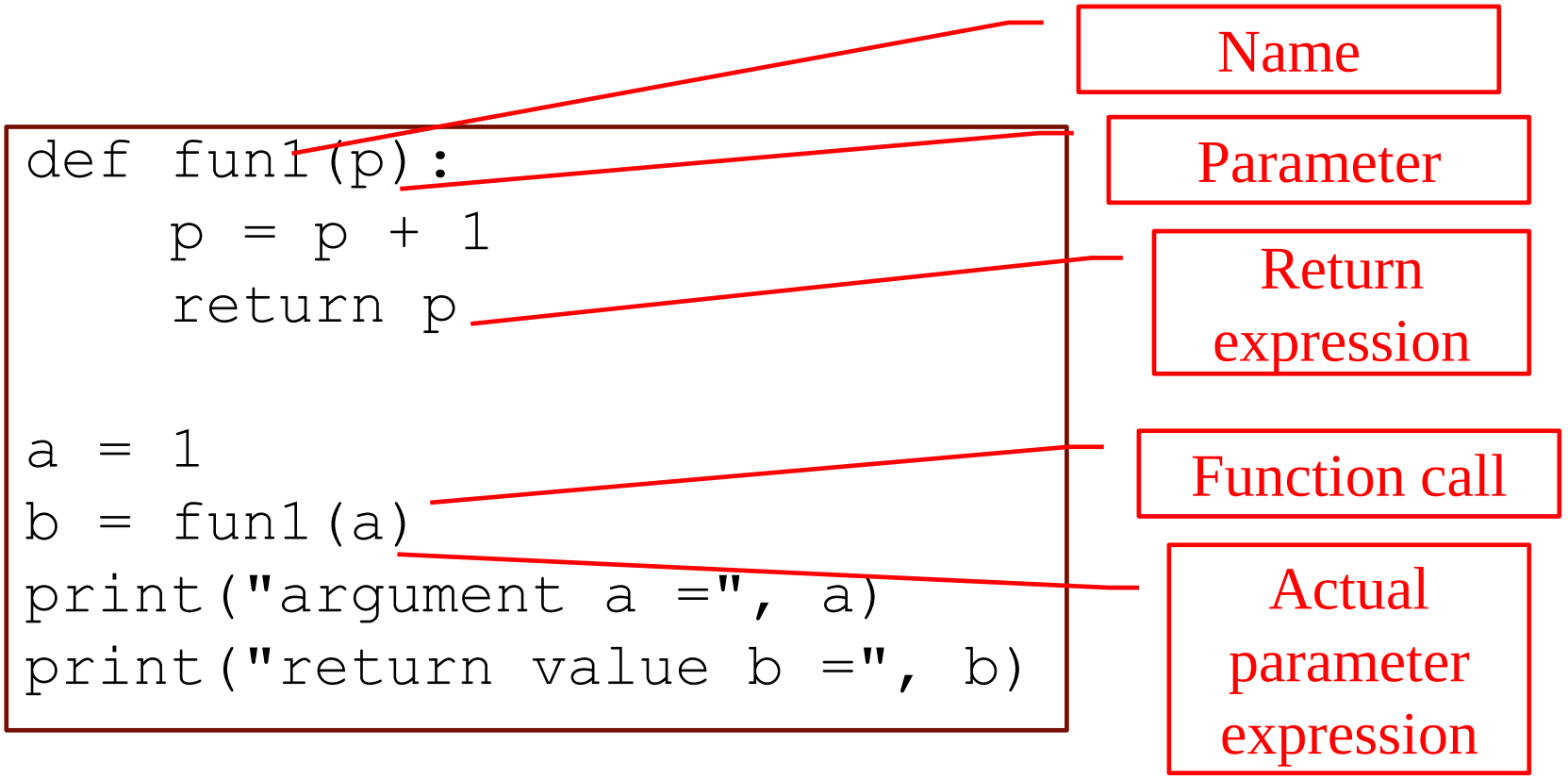
```
def greet():
    print("Hello")
    return
```

```
def greetByName(name):
    return "Hello" + name
```

Parameter & Return

```
def fun1(p):
    p = p + 1
    return p

a = 1
b = fun1(a)
print("argument a =", a)
print("return value b =", b)
```



- What is the result?
- Is variable a changed?

Exercise: 2.1

- Discuss with another teacher
  - Explaining definition versus call
  - Explaining print versus return

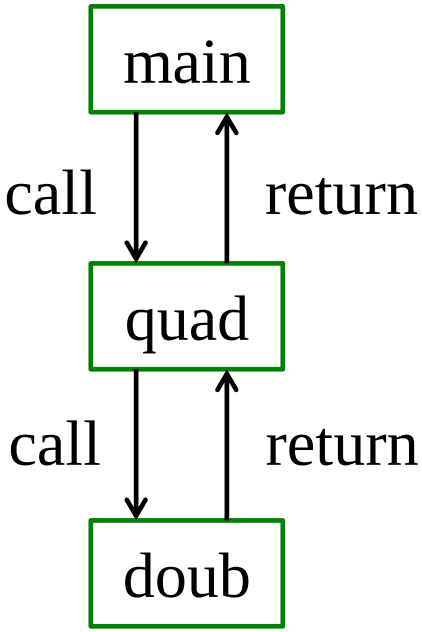
Function Call Tree

- One function calls another

```
def doub(p):
    return p*2

def quad(p):
    d = doub(p)
    return doub(d)

n = int(input("Number: "))
print("4 x", n, "=", quad(n))
```



Function Scope

- Scope: dictionary of variables

```
def doub(p):
    return p*2

def quad(p):
    d = doub(p)
    return doub(d)

n = int(input("Number: "))
print("4 x", n, "=", quad(n))
```

p → integer (6)

p → integer (3)
d → integer (6)

n → integer (3)
doub → … code
quad → … code

Global Variables

- A variable defined in a function is **local**
- Global variables can be read
- Update a global using 'global'

```
def f():
    global g1
    g1 = g1 + g2

g1 = 1
g2 = 3
f()

print("g1 =", g1)
print("g2 =", g2)
```

Update g1

Read g2

Globals instead of parameters
Better to use parameters
Worse to mix

Parameters and Assignment

```
def fun1(thelist):
    thelist.append(41)

myl = [2,3,4]
fun1(myl)
print("myl =", myl)
```

Just like assignment a list:
… variable refers to the list
… parameter refers to the list

No global: reference is read

- What is the result?
- Is the list variable a changed?

Exercise: 2.2-2.3

- Implement ‘shopping list’ functions.

Binary Search Function

```
def BSearch(A, target):
    left = 0
    right = len(A)
    while right > left:
        mid = (left + right) // 2
        if A[mid] == target:
            return mid
        elif A[mid] < target:
            left = mid+1
        else:
            right = mid
    return -1
```

Programming with Recursion

Recursive Algorithms & Programs

- Algorithms described recursively
  - Each stage like the whole
- Can be implemented iteratively
  - Uses a loop
- Can be implemented recursively
  - No loop
  - Recursive function call

Factorial: Iterative and Recursive

```
def factR(n):
    if n <= 0:
        return 1
    else:
        return n * factR(n-1)
```

Recursive call

```
def factI(n):
    f = 1
    for i in range(n,0,-1):
        f = f * i
    return f
```

Exercise: 3.1

- Implement factorial using both recursion and a loop.
- Which do you prefer?

Binary Search (Again)

- Base case:
  - empty array or element found
- Recursive case: search array
  - search either left half or right half of array

```
def findBRec(A, target, left, right):
    ...
    mid = (left + right) // 2
    ...
    return findBRec(A, target, mid+1, right)
    ...
    return findBRec(A, target, left, mid)
```

Merge Sort – Algorithm

```
mergeSort(Array)
    L = length of Array
    if L <= 1 then it is sorted, return it
    else
        mid = L // 2
        A1 = mergeSort(Array from 0 up to mid)
        A2 = mergeSort(Array from mid up to L)
        return merge of A1 and A2
```

- Note: merge sort does not work ‘in place’ – need to copy data items

Merge Sort

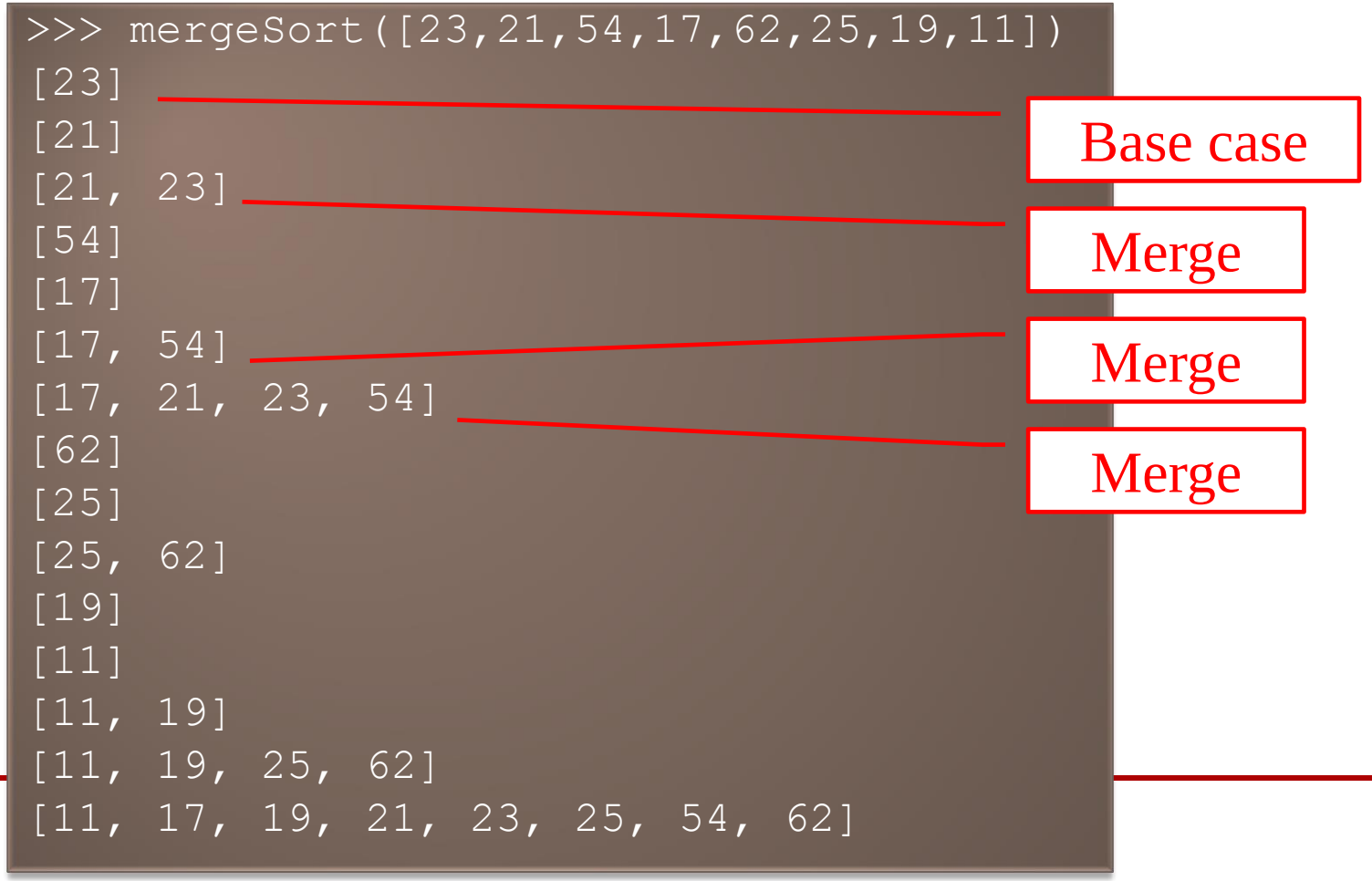
```
def mergeSort(A):
    L = len(A)
    if L < 2:
        return A
    m = L // 2
    A1 = mergeSort(A[0:m])
    A2 = mergeSort(A[m:L])
    return merge(A1, A2)
```

```
def merge(A1, A2):
    if len(A1) == 0: return A2
    elif len(A2) == 0: return A1
    else:
        if A1[0] < A2[0]:
            M = [A1[0]]
            M.extend(merge2(A1[1:], A2))
        else:
            M = [(A2[0])]
            M.extend(merge2(A1, A2[1:]))
    return M
```

Merge Sort Example

- Print sorted lists, base case and after merge

```
>>> mergeSort([23,21,54,17,62,25,19,11])
[23]
[21]
[21, 23]
[54]
[17]
[17, 54]
[17, 21, 23, 54]
[62]
[25]
[25, 62]
[19]
[11]
[11, 19]
[11, 19, 25, 62]
[11, 17, 19, 21, 23, 25, 54, 62]
```



Exercise 3.3

- Complete the recursion implementation of binary search

Summary

Summary

- Recursion
  - Way of thinking
  - Equivalent to loops
- Many algorithms elegantly expressed recursively
- Recursive functions
  - … must understand functions